Name:	
Class:	

YEAR 12

ASSESSMENT TEST 2
TERM 1, 2017

MATHEMATICS

Time Allowed – 120 Minutes
(Plus 5 minutes Reading Time)

General Instructions:

- *All* questions may be attempted
- *All* multiple choice questions are of equal value
- Reference Sheet will be supplied
- Department of Education approved calculators and templates are permitted
- In every Question, show all relevant mathematical reasoning and/or calculations.
- Marks may not be awarded for careless or badly arranged work
- No grid paper is to be used unless provided with the examination paper

The answers to all questions are to be returned in separate bundles clearly labelled Question 6, Question 7, etc. Each question must show your Candidate Number.

ANSWER THE FOLLOWING QUESTIONS, ON YOUR MULTIPLE CHOICE ANSWER SHEET.

1. What is $0.\dot{3}\dot{5}$ as a fraction in its simplest form?

A] $\frac{7}{20}$

B] $\frac{7}{18}$

C] $\frac{35}{99}$

D] $\frac{35}{100}$

2. Differentiate $y = \sqrt[3]{sin^2 6x}$

A] $\frac{2cos6x}{3\sqrt[3]{sin6x}}$

B] $\frac{4\ cos^2 6x}{\sqrt[3]{sin^2 6x}}$

C] $\frac{4cos6x}{3\sqrt{sin^2 6x}}$

D] $\frac{4cos6x}{\sqrt[3]{sin6x}}$

3. Evaluate exactly $\int_1^2 \frac{2x+1}{x+x^2}\,dx$

A] ln3

B] ln12

C] ln6

D] ln4

4. Which line is perpendicular to $3x + 4y + 7 = 0$?

A] $4x + 3y - 7 = 0$

B] $3x - 4y + 7 = 0$

C] $8x - 6y - 7 = 0$

D] $4x - 7y + 7 = 0$

5. For what values of *m* will the geometric series
$1 + 2m + 4m^2 + 8m^3 + \cdots$ have a limiting sum?

A] $-1 \leq m \leq 1$

B] $\frac{-1}{2} \leq m \leq \frac{1}{2}$

C] $\frac{-1}{2} < m < \frac{1}{2}$

D] $m < \frac{1}{2}$

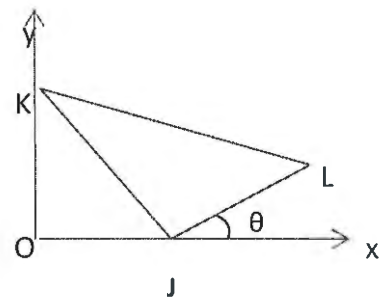
Question Six (27 marks)

(a) Convert 48^0 to radians; give your answer as an exact value. **1**

(b) The points J, K and L have coordinates (1, 0), (0,8) and (7,4).

The angle between the line JL and the x-axis is θ.

Copy the diagram below neatly onto your answer sheet.



(i) Find the gradient of JL **1**

(ii) Find the size of angle θ to the nearest degree. **1**

(iii) Find the coordinates of M, the midpoint of JL. **1**

(iv) Show that JL is perpendicular to KM. **2**

(v) Find the area of ΔJKL **2**

(vi) Write down the coordinates of the point N which makes JKLN a rhombus. **2**

(c) Is $\frac{9}{2}$ a term of the sequence $T_n = \frac{3}{32}(2^{n-2})$? Give reasons. **2**

(d) (i) Find T_n for 14 + 11 + 8 + 5 + ... **2**

(ii) Express 14 + 11 + 8 + 5 + ...+ -34 using Sigma notation. **2**

(e) For the following sequence find the general term in simplest form.

$\log_b 5x^2, \log_b 5x, \log_b 5,$ **3**

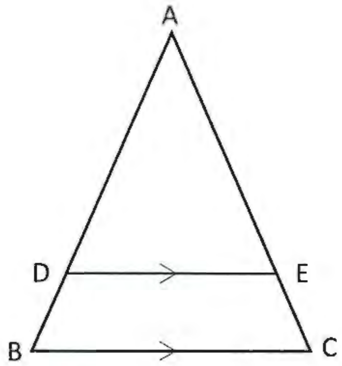
(f) If $S_n = n^2 - 4n$ find an expression for T_n. **2**

(g) In the diagram below; AE = 18cm, EC = 6cm, AB =16cm.

Copy the diagram onto your answer page.

(i) Prove that ΔADE is similar to ΔABC. **2**

(ii) Find the length of DB giving reasons. **2**



(h) Find the size of the angles of the isosceles triangle in which the base angles are double the vertical angle. **2**

Question Seven (27 marks) *START A NEW PAGE*

(a) Draw a possible graph of the curve that satisfies all of the following conditions
$0 \leq x \leq a,\ y'' < 0,\ y' > 0$ and $y > 0$. **1**

(b) On the separate question paper provided, neatly graph $y = f(x)$ and $y = f''(x)$.

Clearly label **each** graph. **2**

(c) (i) Find the coordinates of the stationary points and determine their nature for

the function $y = x^3 - 12x + 5$. **4**

(ii) Find the coordinates of any points of inflexion. **2**

(iii) Neatly sketch $y = x^3 - 12x + 5$ (no need to find x-intercepts). **1**

(d) The area of a sector AOB is $96cm^2$ and its perimeter is 56cm. Find the length of the radius and the size of the angle of the sector to the nearest degree. (O is the centre of the circle). **5**

(e) The number 36 is divided into two parts. The smaller part is multiplied by the square of the larger part.

(i) Show the product is $P = 1296x - 72x^2 + x^3$ **1**

(ii) Hence find the maximum product possible. **4**

(f) If $\frac{dy}{dx} = sec^2 2x$, and y = 0 when x = $\frac{\pi}{6}$ find an expression for y. **2**

(g) Find the exact area enclosed by the curve y = 4cos2x from x = $-\frac{\pi}{6}$ to x = $\frac{\pi}{6}$. **2**

(h) Find the volume of the solid of revolution when $y = e^{x+1}$ is rotated about the x-axis from x =0 to x = ln3 correct to two decimal places. **3**

Question Eight (27 marks) ***START A NEW PAGE***

(a) (i) Show that the first derivative of $y = \ln\sqrt{\frac{1+x}{1-x}}$ is $\frac{dy}{dx} = \frac{1}{1-x^2}$. **2**

(ii) Hence or otherwise evaluate exactly $\int_0^{\frac{1}{3}} \frac{4dx}{1-x^2}$ **2**

(b) Evaluate exactly $\int_0^{\frac{\pi}{8}} (1 - cot4x)dx$ **3**

(c) Show that the exact area bounded by the curve $y = \ln 3x$, the x –axis and the lines $x = \frac{2}{3}$ and x = 1 is given by $A = \frac{1}{3}(3ln3 - 2ln2 - 1)$. **3**

(d) Using Simpson's rule with five functional values approximate the volume of revolution when $y = x\sin x$ is rotated about the x-axis from $x = 0$ to $x = \frac{\pi}{2}$. (Express your answer correct to two decimal places). **4**

(e) A cylinder is inscribed in a sphere of radius 12cm.

(i) If the height of the cylinder is h, show that the volume of the cylinder is given by $V = \frac{\pi h}{4}(576 - h^2)$. **2**

(ii) Find the maximum volume of the cylinder possible. **3**

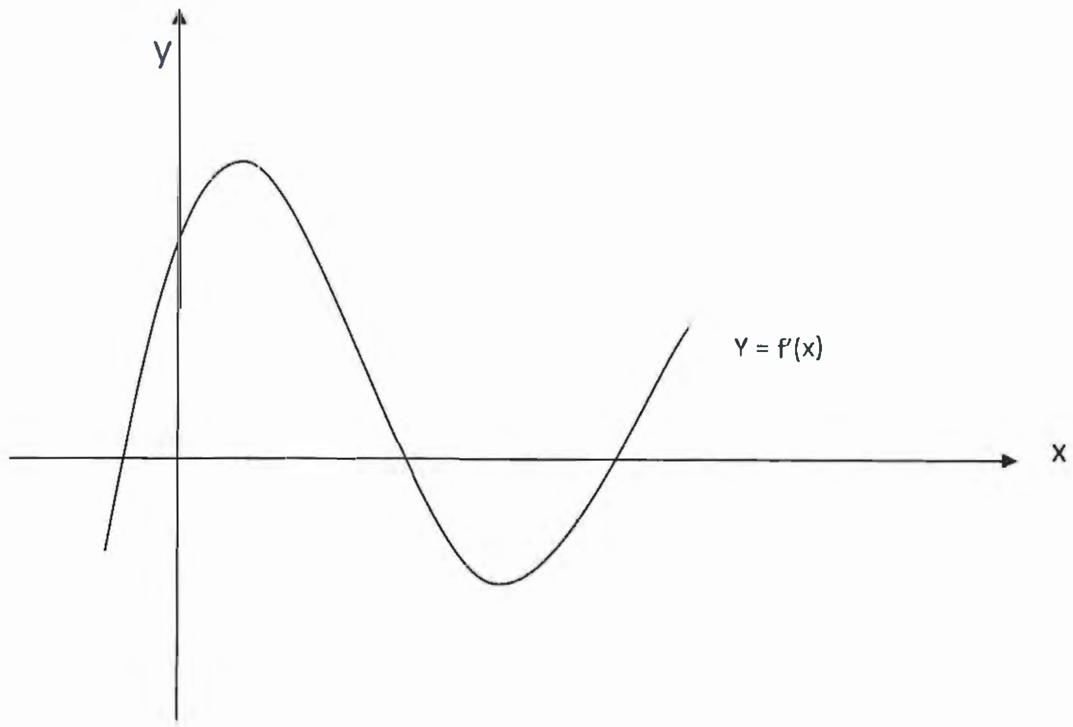
(iii) Find the ratio of this greatest volume of the cylinder to the volume of the sphere. **1**

(f) (i) Graph neatly $y = 3 - 2sinx$ for $0 \leq x \leq 2\pi$. **2**

(ii) How many solutions does the equation $1 - sinx - \frac{x}{3} = 0$ have for $0 \leq x \leq 2\pi$? **2**

(g) Leonard invests \$10 000 into an interest bearing account. At the beginning of every month, starting one month after opening the account, he deposits $\$m$. Interest of 5% p.a. is compounded monthly and is paid at the end of the month. If after 5 years immediately after the interest has been paid his account balance is \$160 000, find the size of the monthly deposit. **3**

End of Paper


y
Y = f'(x)
x

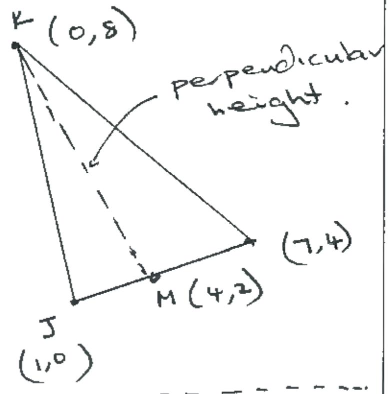
MATHEMATICS: Question M/choice

Suggested Solutions	Marks	Marker's Comments
1. $0.\dot{3}\dot{5} = \frac{35}{99}$	C	
2. $y = \sqrt[3]{\sin^2 6x}$ $y = (\sin 6x)^{2/3}$ $\frac{dy}{dx} = \frac{2}{3}(\sin 6x)^{-1/3} \cdot 6\cos 6x$ $= \frac{4\cos 6x}{\sqrt[3]{\sin 6x}}$	D	
3. $\int_1^2 \frac{2x+1}{x+x^2}\,dx$ $= \left[\ln(x+x^2)\right]_1^2$ $= \ln(2+4) - \ln(1+1)$ $= \ln 6 - \ln 2$ $= \ln 3$	A	

MATHEMATICS: Question M/choice

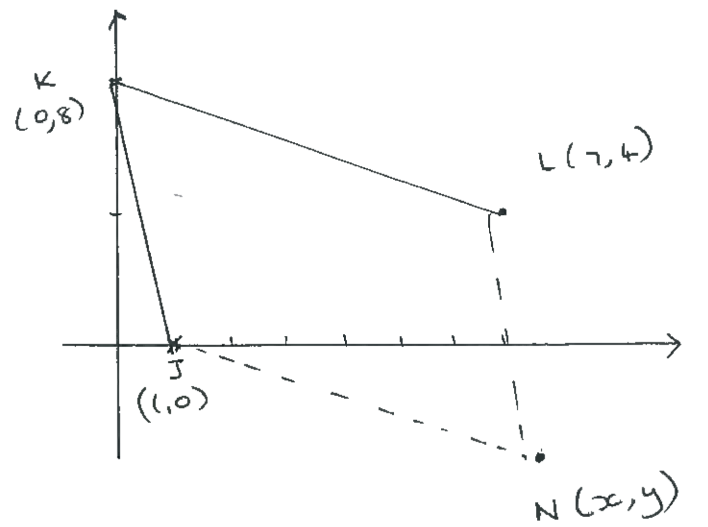
Suggested Solutions	Marks	Marker's Comments
4. $3x + 4y + 7 = 0$ $4y = -3x - 7$ $y = -\frac{3}{4}x - \frac{7}{4}$ $m_1 = -\frac{3}{4}$ $m_2 = \frac{4}{3}$ $y = mx + b$ $y = \frac{4}{3}x + b$ $3y = 4x + 3b$ $0 = 4x - 3y + 3b$ $0 = 8x - 6y + 6b$	C	
5. $1 + 2m + 4m^2 + 8m^3 + \ldots$ $r = 2m$ $-1 < 2m < 1$ $-\frac{1}{2} < m < \frac{1}{2}$	C	

MATHEMATICS: Question 6

Suggested Solutions	Marks Awarded	Marker's Comments
a) $48^\circ = \frac{48\pi}{180} = \frac{4\pi}{15}$ ✓	1 mark.	Generally well done
b) (i) $m_{JL} = \frac{4-0}{7-1}$ $= \frac{4}{6}$ $= \frac{2}{3}$ ✓	1 mark.	
(ii) $\tan\theta = \frac{2}{3}$ $\theta = 33^\circ 41' 24''$ $\theta = 34^\circ$ (nearest degree) ✓	1 mark	Question said to the nearest degree.
(iii) Midpoint of JL $\left(\frac{7+1}{2}, \frac{0+4}{2}\right) = (4, 2)$ ✓	1 mark	
(iv) $M_{KM} = \frac{8-2}{0-4} = \frac{+6}{-4} = -\frac{3}{2}$ ✓ $M_{JL} \times M_{KM} = \frac{2}{3} \times -\frac{3}{2}$ $= -1$ products of gradients is -1 ✓ $\therefore KM \perp JL$	2 marks	1 mark for getting correct gradient of KM 1 mark for proving products of gradients is -1.
(v) Area $= \frac{1}{2} \times d_{JL} \times d_{KM}$ $= \frac{1}{2} \times \sqrt{(7-1)^2 + (4-0)^2} \times \sqrt{(0-4)^2 + (8-2)^2}$ ✓ $= \frac{1}{2}\sqrt{36+16} \times \sqrt{16+36}$ $= \frac{1}{2}\sqrt{52} \times \sqrt{52}$ $= \frac{1}{2} \times 2\sqrt{13} \times 2\sqrt{13}$ $= 26$ ✓ $\therefore$ Area is 26 unit^2	2 marks	 1 mark - Distance JL and perpendicular height 1 mark - formula $\frac{1}{2}bh$

Suggested Solutions	Marks Awarded	Marker's Comments

b (vi)



For rhombus

Distance of JN = Distance of LN

Let N (x,y)

$$\therefore \sqrt{(x-7)^2+(y-4)^2} = \sqrt{(x-1)^2+(y-0)^2}$$

$$\cancel{x^2}-14x+49+\cancel{y^2}-8y+16 = \cancel{x^2}-2x+1+\cancel{y^2}$$

$$0 = 12x+8y-64 \rightarrow eq\ (1)$$

Now $grad_{LN} = grad_{KJ}$

$$\frac{y-4}{x-7} = \frac{8-0}{0-1}$$

$$\frac{y-4}{x-7} = -8$$

$$-8x+56 = y-4 \quad \text{—— eq } (2)$$

$$y = -8x+60$$

Sub eq (2) in (1)

$$12x + 8\cdot(-8x+60) - 64 = 0$$

$$12x - 64x + 480 - 64 = 0$$

$$-52x + 416 = 0$$

$$52x = 416$$

$$x = 8$$

when $x = 8 \Rightarrow y = 60-64 = -4$

$\therefore$ N is $(8,-4)$

Marks Awarded:

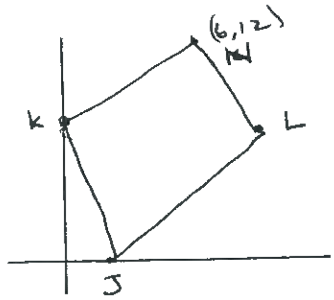
2 Marks

1 mark for working and getting 8

1 mark for substitution to get $y = -4$.

Marker's Comments:

Few students got (6,12) as answer.



You did JKNL and not JKLN

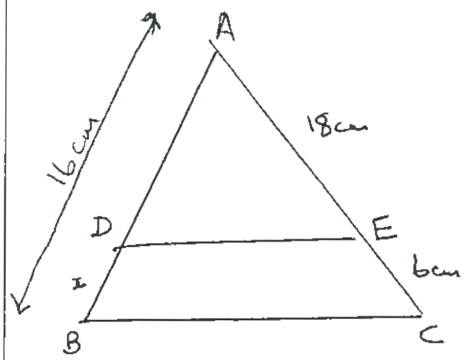
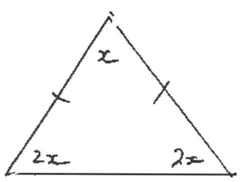
Maximum marks awarded was one.

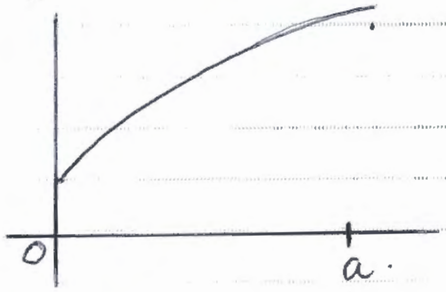
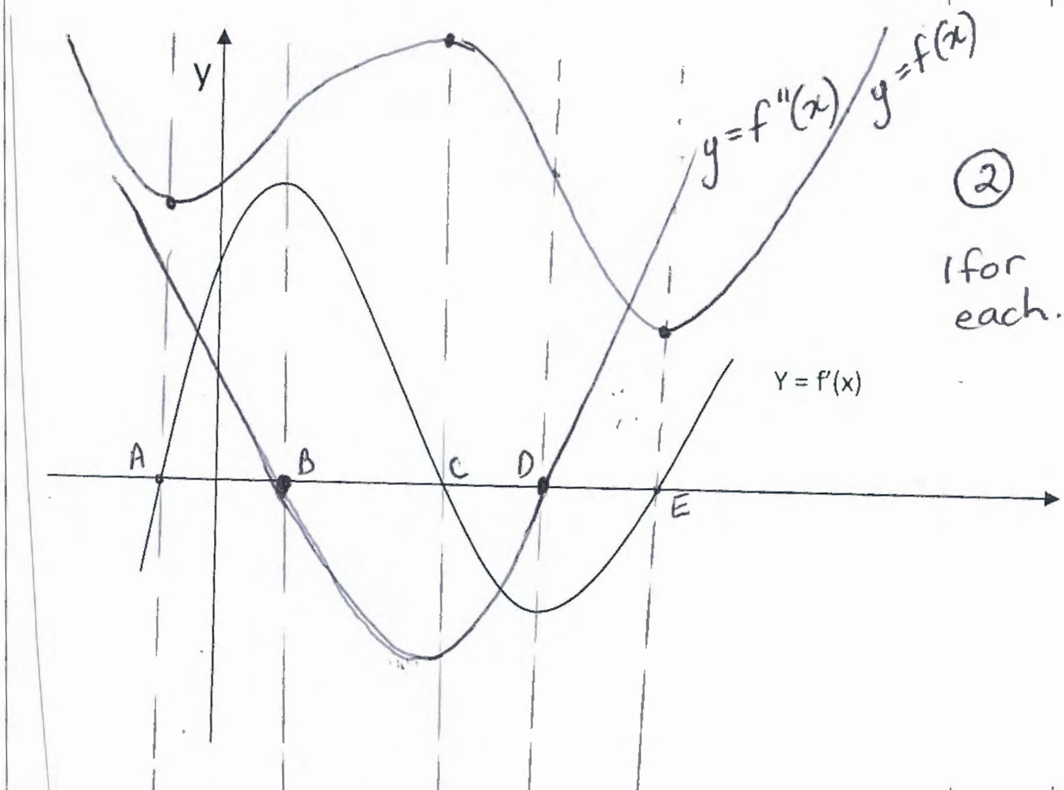
MATHEMATICS: Question…6… page 3		
Suggested Solutions	**Marks Awarded**	**Marker's Comments**
(c) $\frac{9}{2} = \frac{3}{32}.2^{n-2}$ $48 = 2^{n-2}$ $\ln 48 = n-2 \ln 2$. $n-2 = \ln 48 \div \ln 2$ $n = 5.585$ ✓ $\therefore n = 7.8587$ Since n is not an integer $\frac{9}{2}$ is not in the sequence ✓	2 marks 1 mark for substitution and manipulation 1 mark for explanation	OR $\frac{9}{2} = \frac{3}{2}\left(2^{n-2}\right)$ $3 = \frac{1}{16} \times 2^{n-2}$ $3 = 2^{n-6}$ but 2 to no power will give 3 as an answer. * Few students used $2^n - 2$ instead of 2^{n-2}.
(d)(i) $14+11+8+5 - - - -$ $a = 14$ + $d = -3$ ✓ $\therefore T_n = a + (n-1)d$ $= 14 + (n-1)-3$ $= 14 - 3n + 3$ ✓ $= 17 - 3n$	2 marks. 1 for correct substitution of a and d 1 mark for substitution and simplifying	
(ii) $14+11+8+5 \ldots\ldots -34$ $-34 = 17 - 3n$ $3n = 51$ $n = 17$ ✓ $\therefore \sum_{n=1}^{17} (17 - 3n)$ ✓	2 marks 1 mark for getting $n = 17$ 1 mark for summation	accepted $\sum_{n=1}^{17} (14 - 3n + 3)$ as CFE

MATHEMATICS: Question…6… page 4

Suggested Solutions	Marks Awarded	Marker's Comments
(e) $\log_b 5x^2, \ \log_b 5x, \ \log_b 5$ $(\log_b 5 + 2\log_b x), (\log_b 5 + \log x), \ \log_b 5$ It is a AP with $a = \log_b 5x^2$ $d = -\log_b x$ ✓ $T(n) = a + (n-1)d$ $= \log_b 5x^2 + (n-1) - \log_b x$ ✓ $= \log_b 5x^2 - n\log_b x + \log_b x$ $= \log_b 5 + 2\log_b x - n\log_b x + \log_b x$ $= \log_b 5 + 3\log_b x - n\log_b x$ $= \log_b 5 + \log_b x(3-n)$ $= (3-n)\log_b 5x$ ✓ OR $\log_b 5x^{(3-n)}$	3 marks	Poor attempt. 1 mark for recognising $d = -\log_b x$ 1 mark for substituting in $T_n = a+(n-1)d$ 1 mark for final answer.
(f) $S_n = n^2 - 4n$ $S_{n-1} = (n-1)^2 - 4(n-1)$ $= n^2 - 2n + 1 - 4n + 4$ $= n^2 - 6n + 5$ ✓ $T_n = S_n - S_{n-1}$ ✓ $n^2 - 4n - (n^2 - 6n + 2)$ $n^2 - 4n - n^2 + 6n - 5$ $2n - 5$	2 marks	Mostly well done 1 mark for getting S_{n-1} 1 mark for manipulating T_n.

MATHEMATICS: Question…6… Page 5

Suggested Solutions	Marks Awarded	Marker's Comments
g) 	4 marks	Question generally very well done
(i) Prove $\triangle ADE \parallel\!\mid \triangle ABC$ In ADE, $\triangle ABC$ $\hat{A}$ is common $A\hat{E}D = A\hat{C}B$ (corresponding angles are equal as $DE \parallel BC$) $\therefore \triangle ADE \parallel\!\mid \triangle ABC$ (equiangular)	✓ ✓	
(ii) length of DB $\frac{AD}{AB} = \frac{AE}{AC}$ (corresponding sides in similar triangles are in the same ratio) $\frac{16-x}{16} = \frac{18}{24}$ $16-x = \frac{288}{24}$ $\therefore x = 4$ $\therefore DB = 4$ cm.	✓ ✓	reason had to be provided
h) let vertical angle $= x$  $2x+2x+x=180^\circ$ (angle sum of triangle) $5x = 180$ $x = 36$ $\therefore$ angles are $36^\circ, 72^\circ$ and 72°	2 marks ✓ ✓	One had to write all three angles to get full marks.

MATHEMATICS: Question 7 page 1		
Suggested Solutions	**Marks**	**Marker's Comments**
(a) $y''<0$ concave down $y'>0$ increasing $y>0$ above x axis 	1	No half marks. Needed to be concave down + increasing + above axis.
(b) 	2 1 for each.	Look at important features to find $f(x)$. At A: $f'(x)=0$ & $f''(x)>0$ $\therefore$ min TP At B: $f''(x)=0$ & changes sign $\therefore$ POI At C: $f'(x)=0$ & $f''(x)<0$ $\therefore$ max TP At D: $f''(x)=0$ & changes sign $\therefore$ POI At E: $f'(x)=0$ & $f''(x)>0$ $\therefore$ min TP
(c) $y=x^3-12x+5$ $y'=3x^2-12$ $y''=6x$ Possible stationary point when $y'=0$ ie $3x^2-12=0$ $x^2-4=0$ $x=\pm 2$	1	

Suggested Solutions	Marks	Marker's Comments

When $x=2$ $y=8-24+5$
$=-11$

$x=-2$ $y=-8+24+5$
$=21$

$\therefore$ Stationary points at $(2,-11)$ and $(-2,21)$ — 1

To test the nature of the stationary point look at y''

When $x=2$ $y''=12>0$ $\therefore$ concave up
$\therefore$ (relative) min TP at $(2,-11)$ — 1

When $x=-2$ $y''=-12<0$ $\therefore$ concave down
$\therefore$ (relative) max TP at $(-2,21)$ — 1

(ii) Point of inflexion when $y''=0$
AND concavity changes.

ie $6x=0$
$x=0$

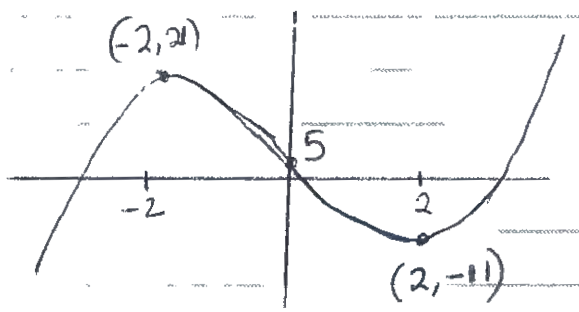
x	-1	0	1
y''	-6	0	6
	<0		$>0.$

1 — for $y''=0$ & $x=0$. $(0,5)$.

1 — for checking

$\therefore$ Change in concavity

When $x=0$ $y=5$
$\therefore$ Point of Inflexion at $(0,5)$



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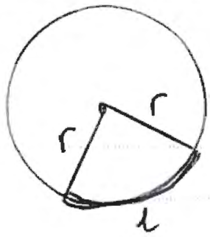
Suggested Solutions	Marks	Marker's Comments

(d)



Area sector $= 96\text{cm}^2$

$A = \frac{1}{2}r^2\theta = 96\text{cm}^2$

$\frac{1}{2}r^2\theta = 96$ — (1)

$r^2\theta = 192$ --- ①

$\theta = \frac{192}{r^2}$

Perimeter $= 56\text{cm}$

ie $l + 2r = 56$ where $l = r\theta$

$r\theta + 2r = 56$

$r(\theta + 2) = 56$ --- ②

Subst $\theta = \frac{192}{r^2}$ - from ①

$r\left(\frac{192}{r^2} + 2\right) = 56$ — (1)

$\frac{192}{r} + 2r = 56$

$192 + 2r^2 = 56r$

$r^2 - 28r + 96 = 0$ — (1)

$(r - 24)(r - 4) = 0$

$r = 24$ OR $r = 4$.

When $r = 4$ $\theta = \frac{192}{r^2} = 12^c$

When $r = 24$ $\theta = \frac{192}{24^2} = \frac{1}{3}$

However, $0 \le \theta \le 2\pi$ because it is a sector of circle $\therefore \theta \ne 12^c > 2\pi$ — (1)

$\therefore \theta = \frac{1}{3}^c$ only.

$\theta = \frac{180}{\pi} \times \frac{1}{3} = 19.0986$

$\doteqdot 19°$. — (1)

$\therefore \underline{\theta = 19°}$ & $\underline{r = 24}$

Marker's comment: Read all the question. You were asked for θ to the nearest degree.

Suggested Solutions	Marks	Marker's Comments
(e) Let the smaller part be x, then the larger part is $36-x$.		Method important - you knew the answer.
$P = x(36-x)^2$		
$P = x(1296-72x+x^2)$		
$P = 1296x - 72x^2 + x^3$	1	
$\frac{dP}{dx} = 1296 - 144x + 3x^2$		
(ii) Possible maximum product when $\frac{dP}{dx} = 0$		Note: If you could not do (i) then use the given result to do (ii). Don't leave it out!!
ie $1296 - 144x + 3x^2 = 0$	1	
$x^2 - 48x + 432 = 0$		
$(x-12)(x-36) = 0$		
$x = 12,\ x = 36 \quad x < 36$	1	
$\therefore x = 12$		
$\frac{d^2P}{dx^2} = -144 + 6x$		
when $x = 12$ $\frac{d^2P}{dx^2} = -144 + 6 \times 12 = -72$		
$\therefore$ concave down	1	
$\therefore$ Max product occurs when $x = 12$ ie $P = 12(36-12)^2 = 6912$.		
$\therefore$ maximum product possible is 6912	1	

Suggested Solutions	Marks	Marker's Comments

(f) $\frac{dy}{dx} = \sec^2 2x$

$y = \frac{1}{2}\int 2\sec^2 2x\,dx$

$= \frac{1}{2}(\tan 2x) + c$ (1)

When $y = 0$ $x = \frac{\pi}{6}$

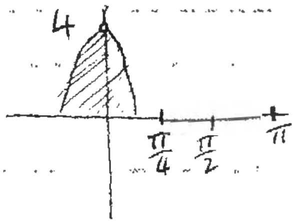
$0 = \frac{1}{2}\tan\frac{2\pi}{6} + c$

$0 = \frac{1}{2}\sqrt{3} + c$

$c = -\frac{\sqrt{3}}{2}$ (1)

$\therefore\ y = \frac{1}{2}\tan 2x - \frac{\sqrt{3}}{2}.$

(g) $y = 4\cos 2x.$



$A = 2\int_0^{\frac{\pi}{6}} 4\cos 2x\,dx$

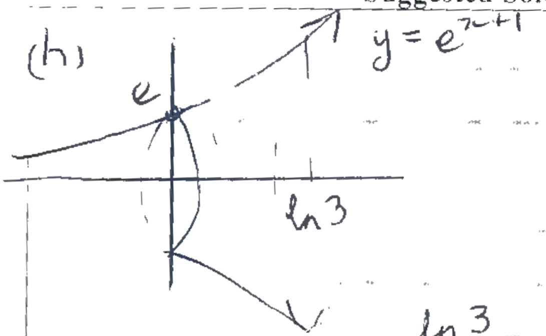
$= \frac{8}{2}\int_0^{\frac{\pi}{2}} 2\cos 2x\,dx$

$= 4\left[\sin 2x\right]_0^{\frac{\pi}{6}}$ (1)

$= 4\sin\frac{2\pi}{6} - 4\sin 0$

$= 4\times\frac{\sqrt{3}}{2} - 4\times 0$ (1)

$= 2\sqrt{3}\,u^2$

Suggested Solutions	Marks	Marker's Comments
(h) 		Volume Formula wrong 0 marks!
$V = \pi \int_0^{\ln 3} \left(e^{x+1}\right)^2 dx$		Note: $\left(e^{x+1}\right)^2 \neq \left(e^x+1\right)^2$
$= \pi \int_0^{\ln 3} e^{2x+2}\, dx$	1	
$= \pi \left[\frac{e^{2x+2}}{2}\right]_0^{\ln 3}$		
$= \pi \left[\frac{e^{2\ln 3+2}}{2} - \frac{e^2}{2}\right]$	1	
$= \frac{\pi}{2}\left(e^{2\ln 3} \cdot e^2 - e^2\right)$		
$= \frac{\pi}{2}\left(e^{\ln 9} e^2 - e^2\right)$		
$= \frac{\pi}{2}\left(9e^2 - e^2\right)$		
$= \frac{8e^2\pi}{2}$		
$= 4e^2\pi\ u^3$		Note if you had CFE, the mark for 2dp was not given unless you had the full calculator answer first.
$= 92 \cdot 85361743$		
$= 92 \cdot 85\ u^3$ (2dp)	1	

MATHEMATICS TASK 2 TERM 1 HSC 2017 1/8

SUGGESTED SOLUTIONS	MARKS AWARDED	MARKERS COMMENT
QUESTION 8)		
a i) $y = \ln\sqrt{\dfrac{1+x}{1-x}}$		
$= \frac{1}{2}\ln(1+x) - \frac{1}{2}\ln(1-x)$	1	
$\dfrac{dy}{dx} = \dfrac{1}{2}\cdot\dfrac{1}{1+x} - \dfrac{1}{2}\cdot\dfrac{1}{1-x}\cdot -1$		common mistake not multiplying (-1)
$= \dfrac{1}{2(1+x)} + \dfrac{1}{2(1-x)}$		
$= \dfrac{1(1-x)+(1+x)}{2(1-x)^2}$	1	
$= \dfrac{1-x+1+x}{2(1-x)^2} = \dfrac{\cancel{2}}{\cancel{2}(1-x)^2}$		
$= \dfrac{1}{(1-x)^2}$ (shown)		
ii) $\displaystyle\int_0^{1/3} \frac{4\,dx}{(1-x)^2}$		
$= 4\displaystyle\int_0^{\frac{1}{3}} \frac{1}{(1-x)^2}\,dx$		
$= 4\left[\ln\sqrt{\dfrac{1+x}{1-x}}\right]_0^{\frac{1}{3}}$	1	
$= 4\ln\sqrt{\frac{4}{3}\times\frac{3}{2}} - \ln\sqrt{\frac{1}{1}}$		
$= 4(\ln\sqrt{2} - \ln\sqrt{1})$		
$= 4\ln\sqrt{2}$	1	
or $4\ln 2^{\frac{1}{2}}$		
$= \ln 2^2 = 2\ln 2$		

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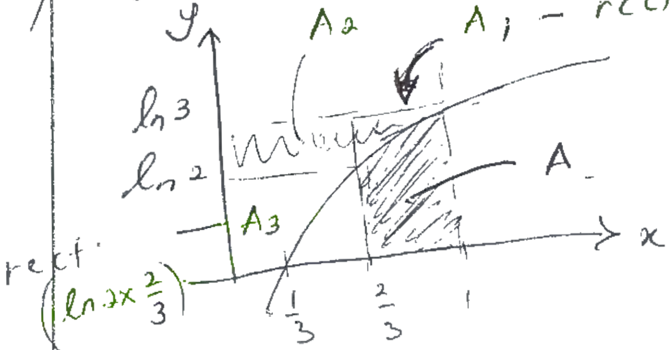
SUGGESTED SOLUTIONS	MARKS AWARDED	MARKERS COMMENT
8b) $\int_0^{\pi/8} (1 - \cot 4x)\, dx$		
$\int_0^{\pi/8} 1 - \frac{\cos 4x}{\sin 4x}\, dx$	1	
$\left[x - \frac{1}{4} \ln(\sin 4x) \right]_0^{\frac{\pi}{8}}$		many students didnt write undefined but instead wrote 0 which is incorrect.
$= \left[\frac{\pi}{8} - \frac{1}{4} \ln(\sin \frac{\pi}{2})\right] - \left[\left(-\frac{1}{4} \ln \sin 4(0)\right)\right]$	1	
$= \frac{\pi}{8} - \frac{1}{4}(0) +$ undefined.		
$= \frac{\pi}{8}$	1	
8c) $y = \ln 3x$		
		
$A_2 = \int_{\ln 2}^{\ln 3} \frac{1}{3} e^y$	1	
$= \frac{1}{3}\left[e^y\right]_{\ln 2}^{\ln 3}$		
$= \frac{1}{3}\left(e^{\ln 3} - e^{\ln 2}\right)$		
$= \frac{1}{3}(1) = \frac{1}{3}$		

SUGGESTED SOLUTIONS	MARKS AWARDED	MARKERS COMMENT
$\therefore A = A_1 - A_3 - A_2$	1	to find A_1 & A_3
$= \ln 3 - \frac{2}{3}\ln 2 - \frac{1}{3}$	1	
$= \frac{1}{3}(3\ln 3 - 2\ln 2 - 1)\,u^2$		

8 d i) $y = x\sin x$. $y^2 = x^2\sin^2 x$.

for 5 func rule from $0 - \frac{\pi}{2}$

x	0	$\pi/8$	$\pi/4$	$3\pi/8$	$\pi/2$
y^2	0	$\frac{\pi^2}{8}\sin^2\frac{\pi}{8}$	$\frac{\pi}{4}^2\sin^2\frac{\pi}{4}$	$\frac{3\pi}{8}^2\sin^2\frac{3\pi}{8}$	$\frac{\pi}{2}^2\sin^2\frac{\pi}{2}$
w	1	4	2	4	1
	0	0.02258	0.30842	1.18466	2.4674

1 — able to write weighing & intervals correctly.

$\therefore V = \pi\int_0^{\pi/2} x^2\sin^2 x\, dx$ — 1 — to write V with π & y is squared

using Simpsons Rule

$= \pi\left[\frac{\frac{\pi}{8} - 0}{3}\left(0 + 0.02258\times 4 + 0.30842\times 2 + 1.18466\times 4 + 2.4674\right)\right.$

1 h correct.

$= \pi\left(\frac{\pi}{24}\right)(7.91321\ldots)$

$= 3.25\,u^3$ (2 dp) — 1

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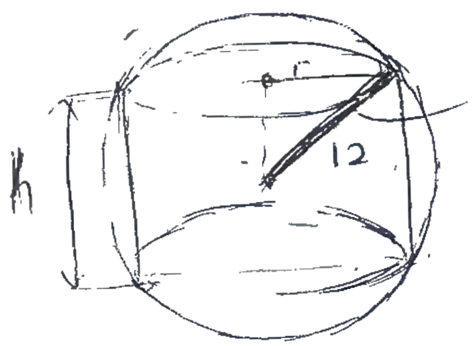
SUGGESTED SOLUTIONS	MARKS AWARDED	MARKERS COMMENT

8 e) i)



radius of sphere.

using pythagoras

$h^2 + 2r^2 = 24^2$ OR $r^2 + \left(\frac{h}{2}\right)^2 = 12^2$

$4r^2 = 576 - h^2$

$r = \sqrt{\frac{576 - h^2}{4}}$

$= \pm \frac{\sqrt{576 - h^2}}{2}$ $r > 0$ — 1

$V = \pi h r^2$

$= \pi h \left(\frac{576 - h^2}{4}\right)$ — 1

$= \frac{\pi h}{4}(576 - h^2)$ shown

ii) To find max

do $\frac{dV}{dh}$ and expand V from above

$\therefore V = 144\pi h - \frac{\pi h^3}{4}$

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SUGGESTED SOLUTIONS.	MARKS AWARDED	MARKERS COMMENT
$\frac{dV}{dh} = 144\pi - \frac{3\pi h^2}{4}$		
$\frac{dV}{dh} = 0$		
$\frac{3\pi h^2}{4} = 144\pi$		
$h^2 = 192$		
$h = 8\sqrt{3}$ $\quad h > 0$	1	
$\frac{d^2V}{dh^2} = -\frac{6\pi h}{4}$		
$= -\frac{3\pi h}{2}$ when $h = 8\sqrt{3}$ $\frac{d^2V}{dh^2} < 0$	1	some students assumed it will be max without doing $\frac{d^2V}{dh^2}$ nor the table no. marks awarded.
$\therefore$ rel max at $h = 8\sqrt{3}$		
OR.		

h	12	$8\sqrt{3}$	14
$\frac{dV}{dh}$	+	0	—
	$\nearrow$	—	$\searrow$

$$\therefore V_{max} = 144\pi(8\sqrt{3}) - \frac{\pi(8\sqrt{3})^3}{4}$$

$$= 1152\sqrt{3}\pi - 384\sqrt{3}\pi$$

$$= 768\sqrt{3}\pi \ u^3$$

1

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SUGGESTED SOLUTIONS.	MARKS AWARDED	MARKERS COMMENT

8 e) iii) Ratio V_{max} Cylinder : V sphere

$$V\text{sphere} = \frac{4}{3}\pi r^3$$
$$= \frac{4}{3}\pi (12)^3.$$

common error - students used $r = 8\sqrt{3}$

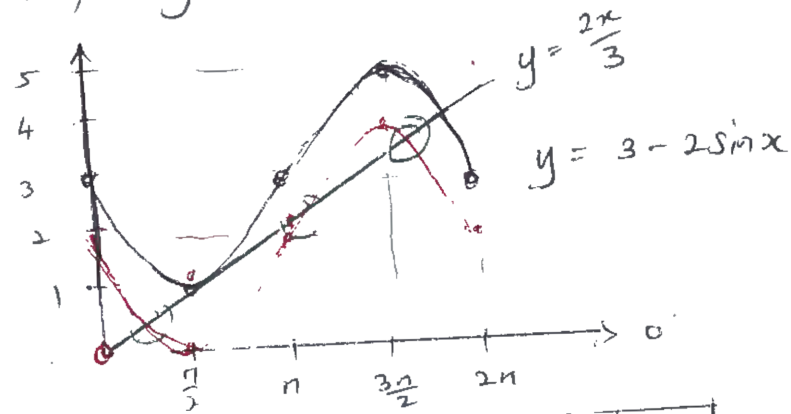
$$\therefore 768\sqrt{3}\cancel{\pi} : \frac{4}{3}\cancel{\pi}(1728)$$
$$\sqrt{3} : 3.$$
or $$1 : \sqrt{3}$$

any of this form is 1 mark.

no marks awarded if written in decimal

f) Graph $y = 3 - 2\sin x \quad 0 \le x \le 2\pi$

i)



1 mark for shape

1 mark for end points.

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$2\sin x$	0	2	0	-2	0
$y = 3 - 2\sin x$	3	1	3	5	3

ii. $$1 - \sin x - \frac{x}{3} = 0$$

$$\left(1 - \sin x = \frac{x}{3}\right) \times 2$$

1 mark

$$\boxed{2 - 2\sin x} = \frac{2x}{3}$$

shift the graph down from $3 \rightarrow 2$ then draw $y = \frac{2x}{3}$

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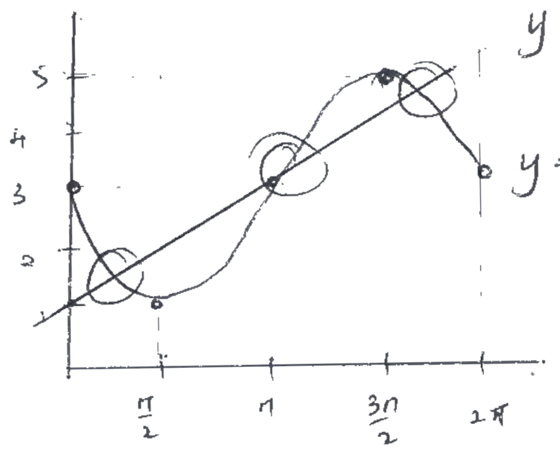
SUGGESTED SOLUTIONS.	MARKS AWARDED	MARKERS COMMENT

OR.

$2 - 2\sin x = \frac{2x}{3}$

$3 - 2\sin x = \frac{2x}{3} + 1$



$\therefore$ 3 solutions — (1) mark

8g)

$A_1 = 10000 \times \frac{241}{240}$

$r = \frac{0.05}{12} = \frac{1}{240}$ but its compounded $\therefore r = \frac{241}{240}$, $n = 60$

(1) mark for writing $A_1, A_2 \to A_{60}$

$A_2 = (A_1 + m) \times \frac{241}{240}$

$= 10000\left(\frac{241}{240}\right)^2 + m\left(\frac{241}{240}\right)$

$A_3 = 10000\left(\frac{241}{240}\right)^3 + M\left(\frac{241}{240}\right)^2 + M\left(\frac{241}{240}\right)$

(1) mark to write in GP form

$A_{60} = 10000\left(\frac{241}{240}\right)^{60} + M\left(\frac{241}{240}\right)^{59} + \dots M\left(\frac{241}{240}\right)$

$= 10000\left(\frac{241}{240}\right)^{60} + M\left[\frac{241}{240} + \left(\frac{241}{240}\right)^2 + \dots \left(\frac{241}{240}\right)^{59}\right]$

$160000 = 10000\left(\frac{241}{240}\right)^{60} + M\left[\frac{\frac{241}{240}\left(\frac{241}{240}^{59} - 1\right)}{\frac{1}{240}}\right]$

$M = \$2196.31$ (nearest cent) — (1)

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SUGGESTED SOLUTIONS.	MARKS AWARDED	MARKERS COMMENT
OR.		
$A_1 = 10000R.$		
$A_2 = 10000R^2 + MR$		
$A_3 = 10000R^3 + MR^2 + MR.$		
$A_n = 10000R^n + M(R + R^2 + \dots + R^{n-1})$		
$= 10000R^n + M\dfrac{R(R^{n-1} - 1)}{R-1}$	$a = R$ $R = R$	$= \dfrac{241}{240}$ $= \dfrac{241}{240}$ $n = 60$
$A_{60} = 10000\left(\dfrac{241}{240}\right)^{60} + M\left(\dfrac{\frac{241}{240}\left(\left(\frac{241}{240}\right)^{59} - 1\right)}{\frac{241}{240} - 1}\right)$		
$= \$2196.31$ (2 dp)		